A CASE STUDY OF CONTRAST INVERSION AND MODULATION TRANSFER RELATED TO THE FINITE X-RAY TUBE FOCAL SPOT SIZE

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Abstract— **The paper presents a case study – explanation of the Contrast Inversion phenomenon in Radiography. This explanation is related to understanding and interpretation of the Modulation Transfer Function (MTF). Its derivation, in the case of blur related to the finite focal spot size of the X-ray tube, is presented as an element of the educational process, which could be used in MTF – related lectures and discussions of artefacts.**

Keywords— **Contrast Inversion, MTF, Image quality assessment, artefacts.**

I. CONTRAST INVERSION MANIFESTATION

One phenomenon which can be seen during Quality Control (QC) tests is Contrast Inversion. It exists in anatomical images, but is difficult to be detected visually. The image on Fig.1 shows a test object (phantom) with its typical resolution pattern. With the increase of spatial frequency, one can clearly observe Contrast Inversion – instead of representing the test object with three cuttings thought the phantom material (represented by three dark lines of the bars in the lower frequencies region), on the Fig.1 image the high frequency patterns are visualized with two dark lines of the bars (as if we have two cuttings only).

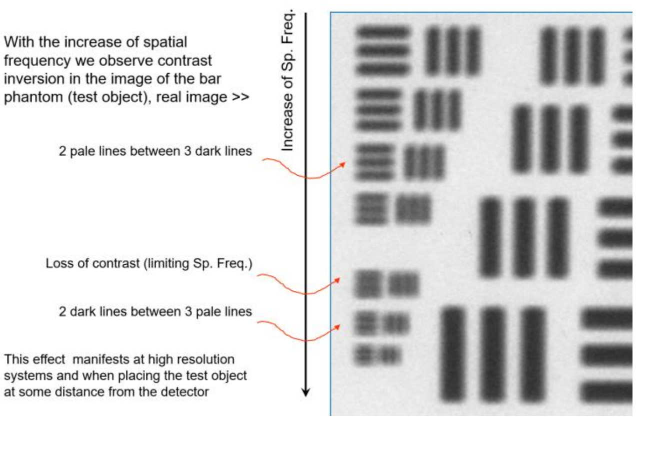

Fig.1 The phenomenon of Contrast Inversion observed in the two highest frequencies patterns/bars of the test object. The right site of the test object image (with low frequencies) is cropped to allow better zoom of the phenomenon (explained on figure: before and after its manifestation – see the arrows).

II. CONTRAST INVERSION EXPLANATION

In order to simplify the explanation of this phenomenon we shall assume that the phantom patterns/bars are not rectangular, but sinusoidal (i.e. with gradually changing attenuation, instead with sharp changing of it). This approximation is often made in MTF discussions.

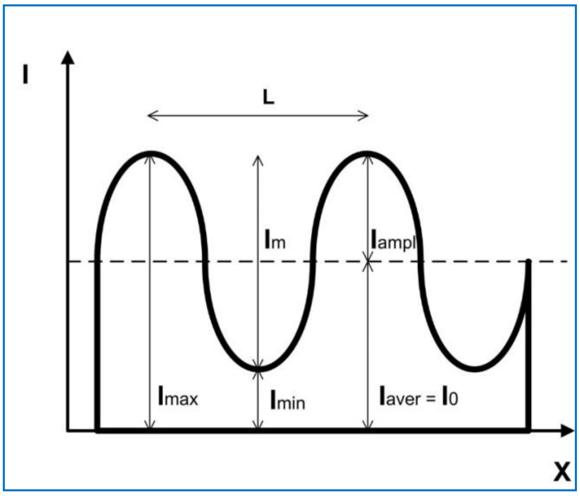

Fig.2 Intensity (I) of the X-rays after their modulation by a hypothetical phantom with the above sinusoidal shape. X is related to the dimensions of the phantom. L is the period of the phantom structures.

Fig. 2 shows part of such a hypothetical sinusoidal test object (phantom) with period L. Equation 1 describes the spatial frequency (ϑ) of this pattern, and also the relation of it with the angular spatial frequency (ω)

$$\vartheta = \frac{1}{L} = \frac{\omega}{2\pi}$$

Eq. 1

Assuming a homogeneous test object, the signal amplitude at any point of the object will depend on its average signal (I_0) plus the change of the amplitude (I_{ampl})

with the angular spatial frequency (ω) - i.e. with the position. Using the intensities shown on Fig.1, we can describe the modulated intensity (I) at a point after the phantom with Equation 2. This way we have the signal (X-ray beam intensity after its modulation by the phantom) separated in two imaginary parts: *fixed* and *variable* (i.e. as if the test object is rectangular block with +/- sinusoidal changes of the shape – hence, the attenuation).

$$I = Io + Iampl\ \sin \omega x = Io + \frac{Im}{2}\ \sin \omega x$$

Eq. 2

Let us place this phantom (test object) in a planar X-ray imaging (radiography) system, where (F) is the size of the Effective focal spot of the X-ray tube. Also, let us assume that we have ideal detector (i.e. no detector blur) – Fig.3

The described imaging system will have magnification (M), depending on the geometry of the system positions: focal spot, object and detector. See from Figure 2 the expression of magnification (M), depending on the distances between focal spot / phantom / detector (A and E).

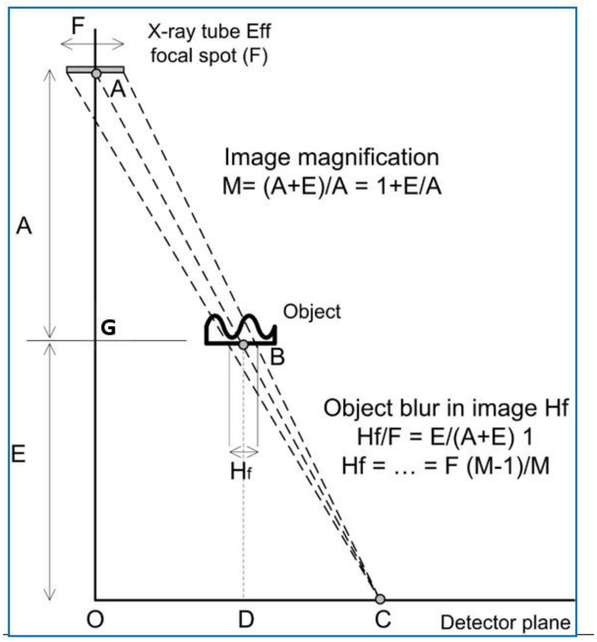

Fig.3 Placing the phantom (object) in a radiographic system. The Effective focal spot size of the X-ray tube has dimension (F) and the detector plane includes an ideal (non-blur) detector.
The expressions on the Figure describe the magnification of the system (M), as well as its influence over the area of the phantom (H_f) projected at point C of the detector.

The focal spot of the X-ray tube (F) is not a point source. It has certain dimension, hence one point of the detector (C) will receive photons from all parts of the focal spot.

The central X-ray beam (from the middle of the focal spot to point C) will pass through point (B) of the object. The spread of B - the irradiated area of the phantom (H_f) - will depend on focal spot size (F). See from Fig.3 the relation between (H_f), focal spot (F) and magnification (M).

Let us observe the similar triangles ACO and ABG on Fig. 3.

In ACO we have: A is the central point of the Focal spot (F); C is the projection of point B from the phantom over the Detector (a composite projection); O is the perpendicular from the central point of the focal spot to the detector. In ABG we have additionally G – the projection of point B over the perpendicular from the focal spot (i.e. the position of the phantom in the system). From these triangles we have:

$$OC = x\ ;\ OD = \frac{x}{M}$$

Eq. 3

Now let us look at the triangle (with dotted lines) made of the whole size of the Focal spot (F) and the projection point over the detector (C), and its similar triangle form by the same point (C) and (H_f) – the irradiated part of the phantom. From these triangles we can express (H_f) as a function of the focal spot size (F) and the magnification of the image (M) – Equation 4

$$\frac{Hf}{E} = \frac{F}{A+E} \quad \gg \quad Hf = F\ \frac{M-1}{M}$$

Eq. 4

Obviously the size of the irradiated part of the phantom (H_f) is directly related to the Focal spot size (F) and the magnification (M) – i.e. position of phantom in the system.

The intensity of the X-rays at point (C) is (I_c). It is related to (I_b) the intensity in point (B), through the inverse square law – i.e. the intensity (I_c) has decreased M^2 times – Equation 5.

Using (Eq. 2) the intensity in point B, (I_b) will be as in Equation 6.

$Ic = \frac{Ib}{M^2}$	$Ib = Io + \frac{Im}{2} \sin \omega x$
Eq. 5	Eq. 6

The intensity (I_b) is actually not in a point, but distributed over the spread (H_f). To describe it, we have to normalize it per unit of length of (H_f), and after this integrate over the length of (H_f). This way the relative change of intensity for the whole (H_f) length will be expressed through an integral from the middle of (H_f) - what is OD from Eq. 3, +/- half of the irradiated area (H_f) - Equation 7

$$Ib = Io + \frac{Im}{2\,\mathrm{Hf}} \int_{\frac{X}{M}-\frac{Hf}{2}}^{\frac{X}{M}+\frac{Hf}{2}} \sin \omega x \, \mathrm{d}\, x$$

Eq. 7

If we further use (Eq.5) to describe (I_c), the intensity in point (C), also using (Eq.7) , we shall have as a final solution of the integral for (I_c) – Eq. 8

$$Ic = \frac{Ib}{M^2} = \frac{Io}{M^2} + \frac{Im}{2\,\omega \mathrm{Hf}\, M^2} \int_{\frac{X}{M}-\frac{Hf}{2}}^{\frac{X}{M}+\frac{Hf}{2}} \sin \omega x \, \mathrm{d}\, \omega x = \ldots$$

$$\ldots = \frac{Io}{M^2} - \frac{Im}{2\,\mathrm{Hf}\, M^2}\left[\cos\omega\left(\frac{x}{M}+\frac{Hf}{2}\right) - \cos\omega\left(\frac{x}{M}-\frac{Hf}{2}\right)\right] = \cdots$$

$$\ldots = \frac{Io}{M^2} + \frac{Im}{\frac{\omega \mathrm{Hf}}{2}\, 2\, M^2}\, \sin\frac{\omega Hf}{2}\, \sin\frac{\omega x}{M}$$

Eq.8

Equation 8 presents the intensity in (I_c), what is in fact the signal getting to the detector from the whole length of the focal spot, after being modulated by the irradiated phantom area in (H_f).

This signal will be “ideal” (without modulation related to fact that the effective focal spot is not a point source) when the size of the focal spot (F) is close to zero - in this case also the spread (H_f) is close to zero. This will affect the *variable* part of the phantom (Eq. 2) - Equation 9:

$$\lim_{\mathrm{Hf}\to 0} \frac{\sin\frac{\omega Hf}{2}}{\frac{\omega Hf}{2}} = 1$$

Eq. 9

This means that, after applying (Eq.8) and (Eq.9) , the maximal signal intensity (I_{cmax}) will be as in Equation 10:

$$\mathrm{Ic\ max} = \frac{Io}{M^2} + \frac{Im}{2\, M^2}\, \sin\frac{\omega x}{M}$$

Eq.10

The Modulation Transfer Function (MTF, or M_f) represents the system modulation – in broad terms: the ratio between the output modulated signal and the input “ideal” signal - i.e. the change of the signal amplitude (per spatial frequency) due to the modulation of the system.

In case of point source Focal spot (assuming all other parameters “ideal”), there will be no influence of the system over the signal due to Focal spot size, hence MTF=1. However the real modulation of the signal, related to Focal spot size influence, will be the ratio of the real signal (I_c) in point (C), and the “ideal” signal, which is equal to the maximal input signal (I_{cmax}) - in the ideal case of (F)=0. Thus dividing (Eq.8) to (Eq.10), we have Equation 11 (the difference is only in the *variable* part of the signal):

$$\mathbf{M}f = \frac{\mathrm{Ic}}{\mathrm{Icmax}} = \cdots = \frac{\sin\frac{\omega Hf}{2}}{\frac{\omega Hf}{2}} = \frac{\sin\frac{\pi Hf}{L}}{\frac{\pi Hf}{L}} = \frac{\sin \pi Uf}{\pi Uf}$$

$$Uf = \frac{Hf}{L} = \frac{F}{L} \cdot \frac{(M-1)}{M}$$

Eq.11

In (Eq. 11) (U_f) is a composite parameter, depending on the focal spot size (F), the magnification (M) – i.e. the place of the object between tube and detector, and the test object period (L) - i.e. spatial frequency. (U_f) is minimal when: (F) is minimal, (M) is minimal (object close to detector) and (L) is maximal (ϑ is min).

Using Eq.11 we can present MTF with a function of an attenuating sine (*sinc* function, or *sinus cardinalis*) – Fig. 4.

Here the changing of the sign (+/-) of the sinc function is in fact Inversion of the Contrast. This way the areas (set of spatial frequencies) A and C will have positive contrast, while the areas B and D will have negative contrast, etc. In fact this change of contrast becomes negligible with the increase of spatial frequency ϑ (due to the very small amplitude of the signal) and in reality, apart from area A, we can only see B and very rarely C (i.e. a well-trained eye could observe up to 2 contrast inversions in case of significant focal spot size and magnification). Also, our visual observation usually cannot detect the small changes of the contrast amplitude inside areas B, C, etc.

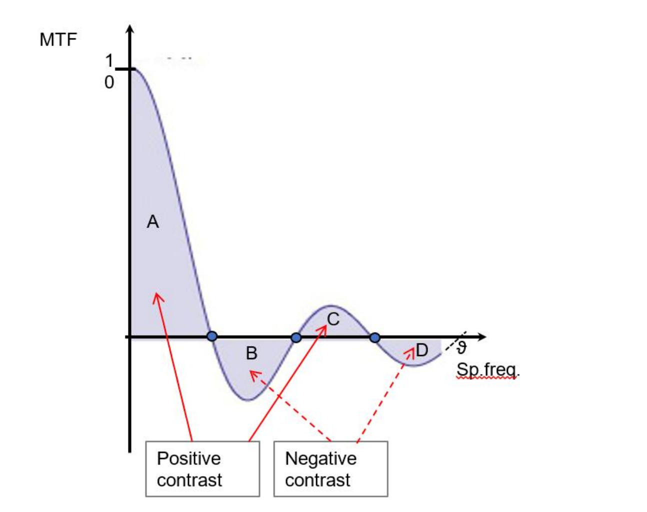

Fig. 4 A real MTF sinc function with contrast inversion areas, visualizing Positive (true) and Negative contrast. At the inflections points the contrast of the object bars disappears, i.e. limiting sp. freq. after area A – see Fig.1

For all image assessments we are normally interested only in area A – where we have the "normal contrast" in the image. Due to this reason the assessment of MTF is limited to only area A, where the Modulation Transfer Function (MTF) has a true meaning - i.e. we use the modulus of the Fourier Transformation (FT) of the Line Spread Function (LSF), or respectively the Point Spread Function:

MTF(f) = | FT{LSF(x)} |

However in reality Contrast Inversion exists – in the presented case study it is due to the blur associated with the geometric size of the Effective Focal spot (or could be from other components of the image system). When the sinc function, associated with the MTF, is presented using the modulus of the Fourier Transformation of the LSF, the modulations after area A (i.e. B, C, D, etc) are presented with positive sign (as if the signal is "rectified"), what may confuse students, unless explained as above.

III. Conclusion

The Contrast Inversion can present a false image of **small object** with larger magnification (i.e. far away from the detector). The image of this object will be with inversed contrast (e.g. pale grey, instead of dark grey and vice versa).

The size of the object, seen with "inversed" image, depends on the size of the X-ray tube effective focal spot and the magnification. This may lead to increase of noise, and what is more important, could mislead the observation of the finding, speaking not for the fact that the pixel values (densities) of such small objects will be completely wrong.

During QC assessment of Image Quality with a Spatial Resolution Test Object (e.g. Hüttner type) the object is usually very close to the detector (i.e. minimal magnification) and due to this reason Contrast Inversion is not observed (unless indicated at the denso-profile – Fig.5). However the anatomical objects within the human body are at different positions – hence with different magnifications. **This may lead to visualizing of small objects, further away from the detector, with inverted contrast.**

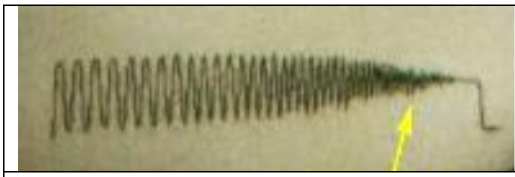

Fig.5 Contrast inversion manifested at the denso-profile of test object with gradually increasing spatial frequencies (plot of pixel values along a line through the image of the test object). The arrow indicates the Contrast Inversion.

The phenomenon Contrast inversion is most obvious when it is related to the blur arising from the finite size of the effective focal spot of the X-ray tube, but it can be related to other "imperfections" of the imaging system. The case study presented here has educational aim, both for students and medical colleagues, while explaining various artefacts or technical reasons for potential misinterpretation of medical images.

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